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Friedmann Dynamics Recovered from Compactified Einstein–Gauss–Bonnet Cosmology

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Abstract—Cosmological dynamics is studied in Einstein–Gauss–Bonnet gravity with a perfect fluid source in arbitrary dimension. A systematic analysis is performed for the case that the theory does not admit maximally symmetric solutions. Considering two independent scale factors, namely, one for the 3D space and one for the extra-dimensional space, it is found that a regime exists where the scale factor of extra dimensions tends to a constant value via damped oscillations for not too negative pressure of the fluid, so that asymptotically the evolution of the $(3+1)$-dimensional Friedmann model with perfect fluid is recovered.

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1. INTRODUCTION

One of the outstanding conceptual features of General Relativity (GR) is that the geometry of space-time itself is a dynamical object. This opens the possibility that there may exist more than three spatial dimensions which are not observable because they may be compactified to a very small scale. This idea was implemented for the first time by Kaluza [1] and Klein [2, 3] assuming one extra spatial dimension in order to attempt to unify gravity with electromagnetism. This idea can be extended to include non-Abelian gauge fields by introducing more extra dimensions. Moreover, the existence of extra dimensions is also predicted by String Theory. The low-energy sector of some string theories is not described by GR but by its generalization known as Einstein–Gauss–Bonnet gravity (see, e.g., [4]). The action of this theory is a sum of three terms, two of which are the familiar Λ-term and the Einstein-Hilbert term whereas the third term is the Gauss-Bonnet invariant which is quadratic in the curvature and reads $I_{\rm GB} = \int \sqrt{-g}(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu})$. In four dimension the Gauss-Bonnet term is topological and does not affect the equations of motion. In dimension higher than four, however, this term leads to a nontrivial contribution to the equation of motion. Since this contribution is still of second order in the derivatives of the metric, Einstein–Gauss–Bonnet gravity can be considered as a natural extension of GR to higher dimensions as its action is constructed according to the same principles as the Einstein-Hilbert action in four dimensions. Einstein–Gauss–Bonnet (EGB) gravity is actually a particular case of a more general gravity theory known as Lovelock gravity [5]. In the same way as the Gauss-Bonnet term, which is topological in four dimensions but gives a non-trivial contribution to the equations of motion in higher dimension in every odd dimension, it is possible to add a new higher-power term in the curvature which is topological in the lower even dimension (this means, for example, that in 7D the most generic Lovelock theory is a sum of four terms, the three terms of Einstein–Gauss–Bonnet gravity and a new term which is cubic in the curvature). All these higher-power terms again lead to a non-trivial contribution to the equations of motion which is of second order in the derivatives.

Even if Lovelock gravity is constructed according to the same principles as GR, it has some features which are absent in GR. For example, in the first-order formalism the equation of motion do not imply vanishing of torsion [6], which therefore becomes a new propagating degree of freedom. Exact solutions

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